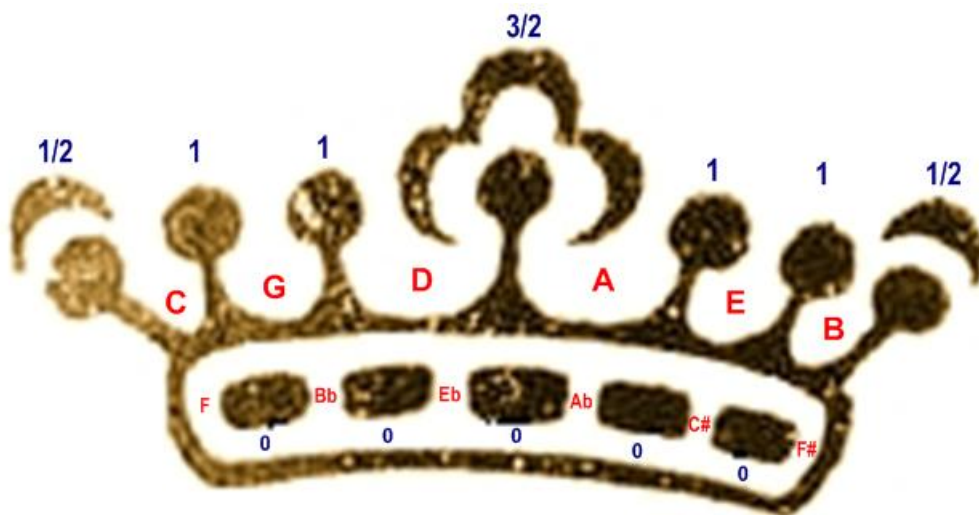




Tuning Interpretation of Bach's '1722 Seal' as Beats Per Second

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Introduction

The Bach seal above [1] has been the subject of previous tuning interpretations. The first by Herbert Anton Kellner, was elaborated in many scholarly publications [2-56] and has been installed on over 300 organs worldwide; numerous Bach recordings have been released in the 'Kellner' temperament. More recently, an alternative tuning interpretation has been offered by Andreas Sparschuh [57]. In this paper, a new approach in terms of beats-per-second is outlined. The method has previously been applied to designs appearing on the cover sheet of J. S. Bach's *Das Wohltemperirte Clavier* [58] and Freiderich Suppig's *Calculus musicus* and *Labyrinthus musicus* [59]. The first known use of the Bach seal (Figure 1) occurred in 1722 [60].



Figure 1: First Known Use of Seal (1722)

The '1722 seal' may be viewed as a depiction of intervals within a contiguous range of twelve semitones. In the lower part of the 'crown', five relations between sharps and flats may be intimated: F-Bb, Bb-Eb, Eb-Ab, Ab-C# and C#-F#; these intervals are interpreted as pure fifths. At the respective ends, F-C and F#-B are slightly tempered such that they beat slowly once in two seconds. The remaining intervals beat more rapidly at once per second, with the exception of D-A which beats more quickly still at three beats in two seconds. As harmonic analysis shows, the tuning is implied for the *small octave*.

A 'canonical' tuning procedure within the small octave may be as follows:

1. Tune five pure intervals F-Bb, Bb-Eb, Eb-Ab, Ab-C# and C#-F#
2. Tune a slowly beating F-C (one beat in two seconds)
3. Tune a slowly beating F#-B (one beat in two seconds)
4. Tune C-G and G-D to beat once per second
5. Tune B-E and E-A to beat once per second
6. Check that D-A beats three times in two seconds

Fourths may be inverted upwards as fifths, if preferred. Fourths or fifths may be transposed up an octave if desired, but must then beat twice as fast.

Derivations

Based on the note names and numerical assignments shown above, the following harmonic equations capturing beats can be defined:

FC:	$3f_5 - 4f_0 = 1/2$
CG:	$3f_0 - 2f_7 = 1$
GD:	$3f_7 - 4f_2 = 1$
DA:	$3f_2 - 2f_9 = 3/2$
AE:	$3f_9 - 4f_4 = 1$
EB:	$3f_4 - 2f_{11} = 1$
BF#:	$3f_{11} - 4f_6 = 1/2$
F#C#:	$3f_6 - 4f_1 = 0$
C#Ab:	$3f_1 - 2f_8 = 0$
AbEb:	$3f_8 - 4f_3 = 0$
EbBb:	$3f_3 - 2f_{10} = 0$
BbF:	$3f_{10} - 4f_5 = 0$

Solving the equations shows that the tuning occurs in the Cammertone small octave c-b (A=415.882 Hz). The temperament is shown in Table 1 and the width of thirds in Table 2.

Note	Cents
C	0.0000
C#	91.9662
D	196.1500
Eb	295.8760
E	391.0500
F	499.7860
F#	590.0110
G	697.3030
Ab	793.9210
A	891.8720
Bb	997.8310
B	1089.2900

Table 1: Temperament

Root	Minor 3rd	Major 3rd
C	296	391
G	301	392
D	304	394
A	308	400
E	306	403
B	307	407
F#	302	408
C#	299	408
Ab	295	406
Eb	294	401
Bb	294	398
F	294	392

Table 2: Width of Thirds in Cents

Comparisons

A comparison of results with previous interpretations is given in Figure 2 and Figure 3

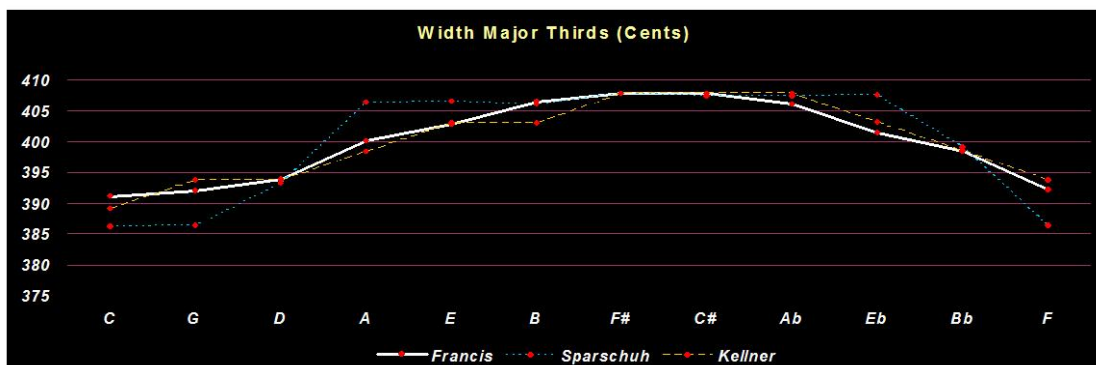


Figure 2: Comparison of Major Thirds

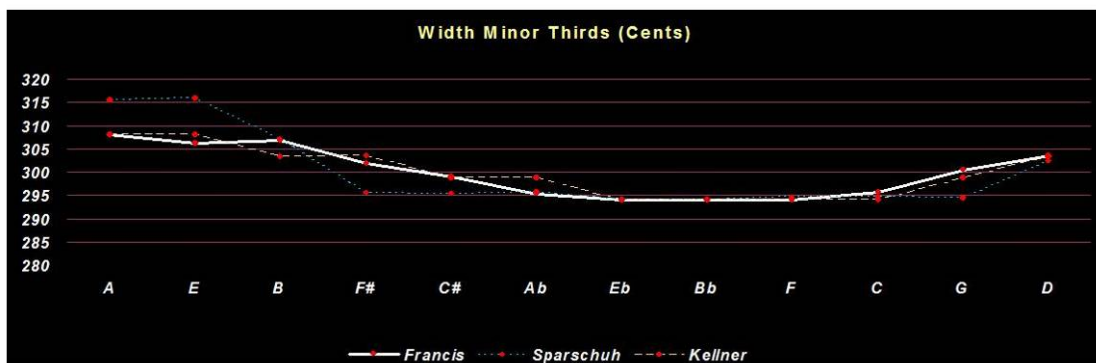


Figure 3: Comparison of Minor Thirds

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